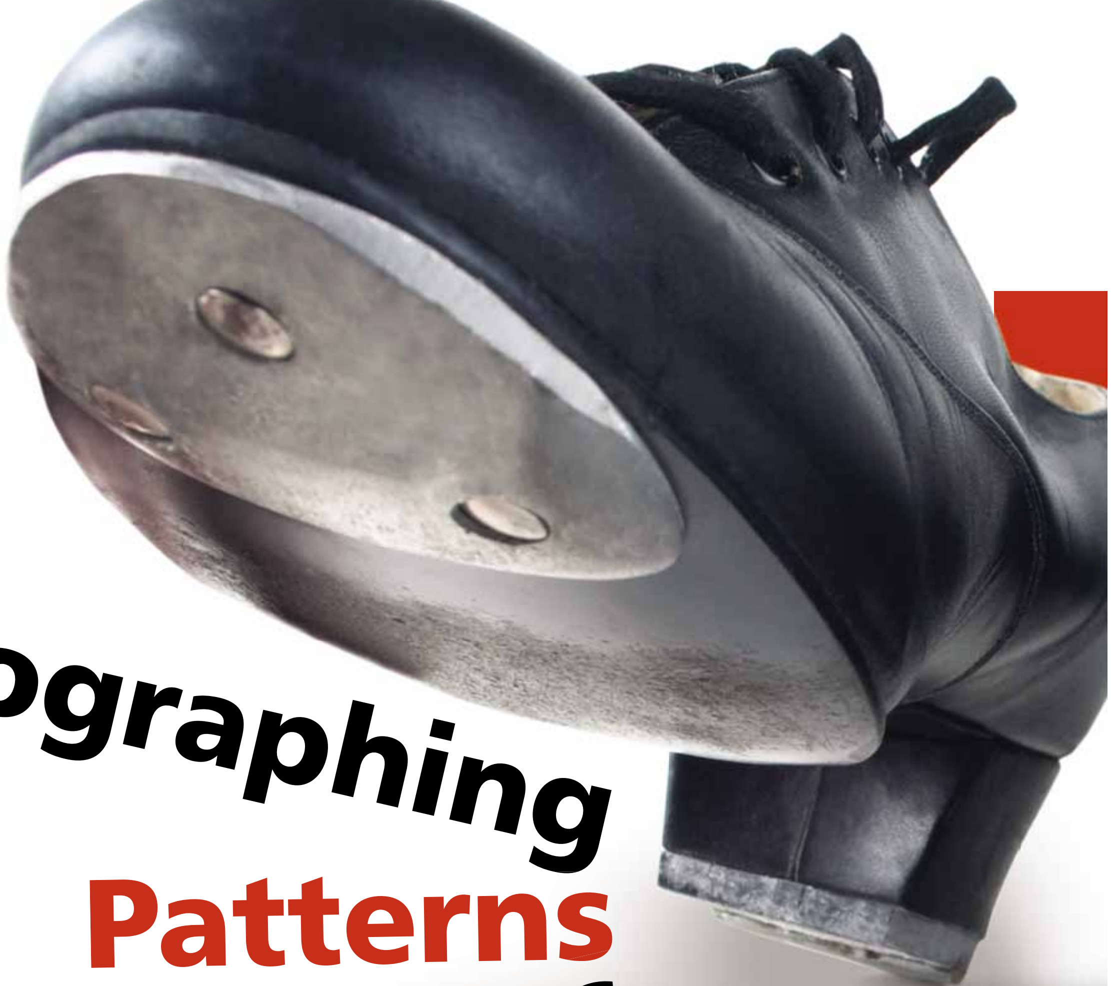
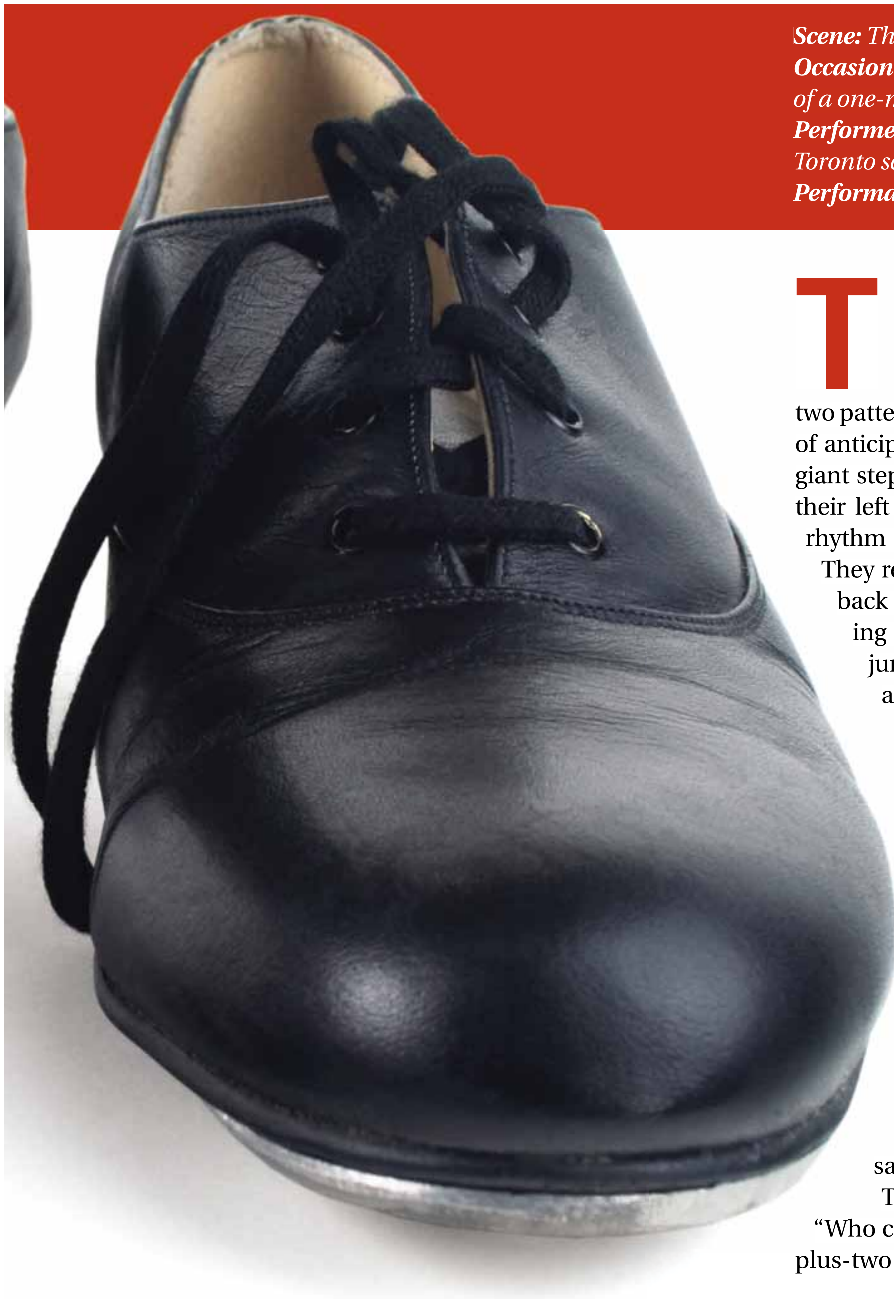


Choreographing Patterns and Functions

By Zachary Hawes, Joan Moss, Heather Finch, and Jacques Katz

Young students dance their way through a multifaceted differentiated approach to early algebra instruction.



Scene: *The school auditorium*
Occasion: *A dance performance celebrating the culmination of a one-month patterning and algebra unit*
Performers: *Members of a grade 1–2 split class in an urban Toronto school*
Performance title: *"Guess My Rule"*

The curtain opens. Audience members turn their attention to center stage, where a pair of first graders are prepared to dance $y = 3x + 2$, less formally known to the dancers as a "times three plus two pattern." The two students exchange a smile and a look of anticipatory excitement, a signal to begin. They take a giant step to their left, bending at the hips and extending their left arm to the side in wave-like motions. There is rhythm and fluidity in their choreographed movements. They repeat the movement, first to their right and then back to their left. Then they take a deep breath, jumping high into the air, performing back-to-back stag jumps. The dancers pause, communicating to the audience that they have completed the first position ($x = 1$) of their dance. They go on to perform the next two positions ($x = 2$ and $x = 3$), multiplying the number of lateral movements (x) by three (the coefficient) but maintaining the number of stag jumps (the constant). The performers exercise both their minds and bodies in transforming an abstract algebra formula into aesthetically pleasing movement, and the audience members are not passive observers. Rather, they are actively engaged in the challenge of guessing the algebraic function that determines the number of movements the dancers perform.

"I think it's a times-three-plus-two pattern," says Brooke. Her peers nod in agreement.

Their teacher, Mr. Katz, prompts them further, "Who can explain how you know that it's a times-three-plus-two pattern?"

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Even typically low-achieving students demonstrated the confidence and competency to design, discern, and dance two-step algebraic functions.

Shaun offers his best explanation, "They started off by … [*pausing and then standing up*] … doing three of these [*demonstrating the lateral movement performed by the dancers*] and then two of these" [*demonstrating the stag jumps*]. Shaun continues to explain how the lateral movements increase by a multiple of three as the number of stag jumps remains constant.

Katz probes further, "Here's a challenge: How would they dance the tenth position?"

After a moment of reflection, students' hands begin to shoot up, and Prateem excitedly responds, "There would be thirty waves and two jumps, because ten times three is thirty and the two just stays the same."

The performances continue as other students engage their peers with their own algebraic choreography. Students take their final bows and exit the stage, excited and proud of their accomplishments. Indeed, the level of understanding of two-step function rules revealed in this vignette would be impressive even for much older students (Stacey and MacGregor 1999). How did this understanding develop? How did such young students arrive at an understanding of the relationship between the independent variable (position, or term, number) and the dependent variable (number of elements or dance moves)? The algebraic dance described above—the translation of composite linear growing patterns into choreographed movement—was the last component of a research-based instructional unit that focused on fostering an understanding of linear functional rules through geometric growing patterns and function-machine activities (see Moss and McNab 2011). The dance component, a late addition to our research project, was both motivating and instructional. All students, including those who were typically low achievers, demonstrated through dance the confidence and competency to design and discern two-step algebraic functions.

Setting the context

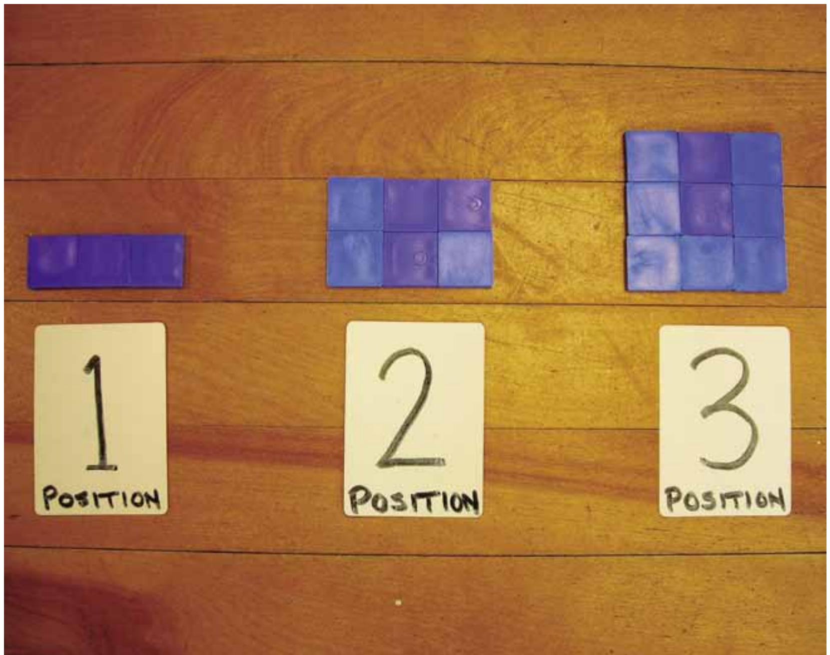
This article gives details of algebraic dance instruction along with illustrative lessons from the research lesson sequence. But first, we present a brief description of the classroom environment in which these lessons took place, as well as a brief overview of the developmental and pedagogical theories that underpin our work.

The classroom

The unit was taught in a combined grade 1–2 classroom in an urban Toronto public school as part of an ongoing algebra research project that has so far involved more than twenty inner-city classrooms. The students in this particular class were the youngest to participate in the project to date and had been described by their teacher as an exceptionally diverse group of learners with a wide range of abilities. Working alongside the classroom teacher, we—the authors of this article—implemented a one-month patterning unit that consisted of eleven lessons that were embedded during the regularly scheduled math periods (see the **online appendix**).

Theoretical framework

The research lessons were designed to support students' discovery of rules for patterns in algebraic functions. Although patterning is ubiquitous in elementary school mathematics programs, a significant body of research suggests that the route from patterns to algebraic rules is challenging for students (Lannin 2005; Stacey and MacGregor 1999). Although students are adept at extending both repeating and growing patterns to "next" positions using a recursive strategy (i.e., adding on to the previous position), they often display difficulty making predictions for elements far down the sequence of growing patterns; they also have difficulty finding rules to express how the pattern is generated (Noss, Healy, and Hoyles 1997). Thus, our instructional sequences were designed to move students beyond a "what comes next" approach, or "recursive strategy," toward a deeper understanding and ability to determine and generalize the functional rules of growing patterns—central to algebraic reasoning (Rubenstein 2002). Our general approach to these objectives is to engage students in activities in which they move back and forth between discerning and developing rules for both geometric and numeric patterns.



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To circumvent students' inclination toward recursive reasoning, use position cards to highlight the relationship between the position number and the number of tiles.

Pedagogy for differentiating instruction

Our approach to instruction and learning is founded in the research and theoretical frameworks that emphasize key elements of differentiated instruction. To this end, we include multiple opportunities for student choice, peer tutoring, game-like learning, multimodal representations, and ongoing assessment as guides to developmentally appropriate instruction (Bray 2009; Gardner 1993; Ministry of Ontario 2008; Small 2009). Additionally, throughout the unit, students transition among whole-group, small-group (both homogeneous and heterogeneous grouping), and individual instruction and learning opportunities. As we demonstrate in the illustrative lessons below, our pedagogical approach fosters a climate of classroom learning in which all children are free to take risks, to work within their own comfort level and, more important, to experience success in their learning.

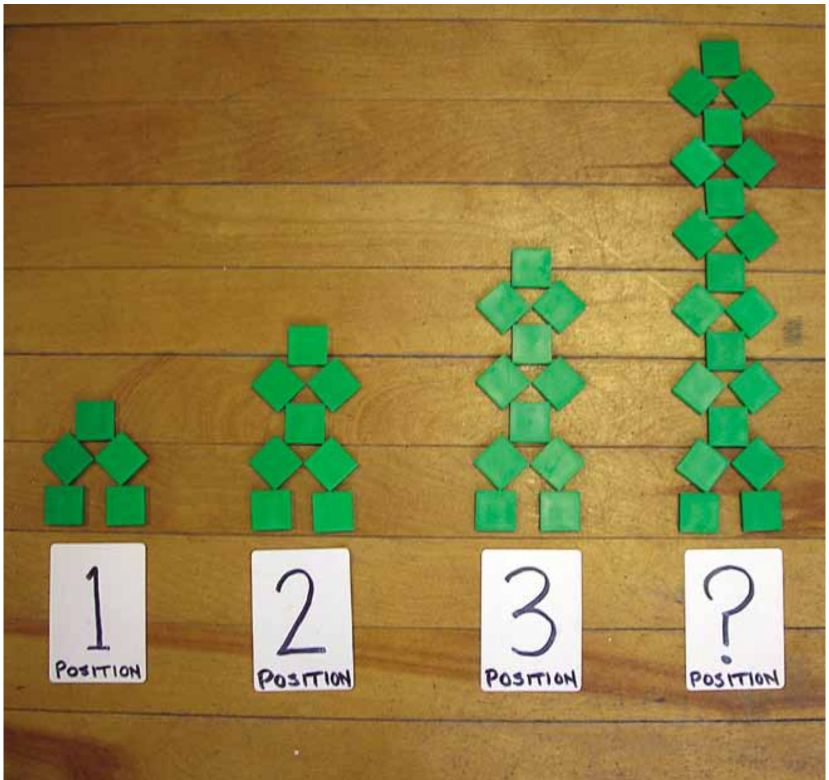
Visual representation: Geometric growing patterns

In all of our research classrooms, the introduction to growing patterns begins in the same way: Students sit in a circle on the floor as the teacher presents the first three positions of a geometric growing pattern made of square tiles set out in arrays that increase by a row of three squares in each position. Specially made "position cards" are placed below the geometric arrays to enable students to keep track of the ordinal position number of these tile patterns. The teacher's questions unfold in a particular way that helps students focus on the relationship between the position number (card) and the number of elements in the corresponding position, thus circumventing students' natural inclination to use recursive reasoning.

To begin this session, Katz asks, "If this pattern keeps growing in the same way, what would the fourth position look like? How many blocks would there be in that position?"

Although the students in this class had not had any formal instruction in multiplication and many of them still struggled with basic arithmetic, they were all able to correctly describe the next position. Student responses naturally varied and revealed important differences in their understanding and approach to the problem:

- Using a recursive strategy, Julia explained that the answer is "twelve because you just add on three each time."
- Natesh's reasoning was grounded in both the visual and numerical features, pointing out



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From gallery walks around the classroom, students try to guess peers' rules about student-designed geometric patterns.

that the position tells you how many rows of three there should be.

- Hannah used a skip-counting strategy to determine the fourth position, explaining, "I think it's twelve because [position] 1 has three [tiles], 2 has six, 3 has nine, and 4 has twelve."

After the class offered and discussed conjectures for the fourth position, Katz—rather than asking for numbers in the fifth position—moved his hand to the far right of the pattern and emphatically revealed a card labeled "position 10." He placed the card down and challenged the class: "What about the tenth position? What would this position look like, and how many tiles do you think there would be? Any ideas?"

Again, student responses were varied and revealing:

- "It would look way bigger."
- "It would be ten rows of three."
- "Well, it's going to be ten up and three to the side" [*motioning*].

With further prompting, students arrived at the total number of tiles:

- "Oh, it's thirty—because three groups of ten [*pointing to the array*] is thirty."
- "Look, ten, twenty, thirty" [*pointing and motioning down the three rows of ten*].

Kevin, a second-grade student, articulated the functional relationship profoundly, "The position card tells you the number of times you have to count to three, which is ten, and I know that ten times three equals thirty, so that's the answer."

By skipping the fifth position and moving directly to the tenth position, Katz helped students avoid "what-comes-next" reasoning and instead focused their attention on the functional relationship between the position number and the number of elements in that position—a central goal of this instruction. After we modeled this same instructional approach with five as the new coefficient ($y = 5x$), students worked in pairs to create their own geometric patterns. Each pair received approximately fifty tiles of uniform color as well as cards labeled position 1, position 2, and position 3. Students chose their own pattern rule (e.g., "a times-three pattern") and worked together to build the first three positions. Naturally, students chose to work with numbers and multipliers within their zone of proximal development. After they finished building their patterns, they went on gallery walks, taking turns touring the room and guessing the algebraic rule that determined their peers' geometric patterns. Students found this part of the lesson particularly motivating. Knowing that their peers were going to be analyzing their patterns in search of an algebraic rule encouraged them to build at their utmost capacity and, furthermore, to be faultless in executing their design and mathematical calculations.

The teacher offered a further challenge: the opportunity for the pattern designers to challenge fellow students to guess a "mystery position," which refers to an example of the same pattern at a position farther down the sequence. The mystery position not only afforded an additional challenge for this diverse group but also helped focus students' attention (both builders' and rule guessers') on the pattern's functional rule (Moss and McNab 2011).

Numeric representations: Function machines

After students had several lessons on designing and discerning rules for one-step geometric growing patterns, we introduced function-machine activities. Typically, these activities involve variations of a Guess My Rule game in which students are challenged to determine function rules by examining the relationship between paired input and output numbers (Carraher and Earnest 2003; Rubenstein 2002; Willoughby 1997). Although this activity has been used in elementary school classrooms before, our research explicitly highlights the connection between the idea of rules in the context of geometric growing patterns and rules governing the relationship between input and output numbers. To reinforce this connection, following the introductory function-machine lessons, we interspersed geometric growing-pattern activities with function-machine activities—first interweaving lessons incorporating one-step patterns and then, later, two-step patterns.

The approach to our function-machine activities involves preparing corresponding input and output numbers on cue cards. The teacher places the input number into the "machine," and an assistant on the other side of the machine then feeds back the output number. After observing two or three pairs, the children are challenged to consider the function rule by predicting the last output number in the series. To help students better "see" the relationship between the nonsequential input and output numbers, the teacher models how to use a t-chart to record the pairs of input and output numbers. Presenting nonsequential pairs of input and output numbers requires students to focus on the "across" rule (on a t-chart)—or function rule—rather than the "down" pattern, or "what-comes-next" strategy.

The youngsters in this research classroom had no previous formal instruction in multiplication, so our initial challenges involved addition (e.g., $y = x + 2$); however, once students became familiar with the Guess My Rule structure, we introduced them to simple multiplicative rules, beginning with pairs of numbers exemplifying the rule $y = 2x$. After being shown two sets of cards—input 5/output 10; input 2/output 4—students were presented with input 10 and had to predict the output. They had little difficulty recognizing that the output would be 20. However, they demonstrated differentiated understanding of what it means to multiply a number by two, which was apparent in their range of responses:

- "I know the number is twenty because you plus the number you have."
- "The number adds on to itself: five goes to ten, two goes to four, and so ten goes twenty."
- "It's just doubles."
- "The number gets bigger by the number you put in."

These students revealed a range of intuitions about the multiplicative process, despite never having been formally introduced to it.

Following several teacher-led examples, students worked in pairs to create their own function-machine challenges. As in the geometric pattern lessons, students had free choice of numbers they selected. They found this activity highly motivating, as they were eager to play the role of "function machine" and stump

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Using a function "machine" encourages the class to guess a functional rule; for example, $y = 2x$, or a times-two rule.

their classmates. Audience members were equally engaged as they filled in t-charts and tried their best to guess the rules.

Kinesthetic representations: Algebraic dance

Once the children had demonstrated a solid understanding of linear functional rules and could both discern and design pattern rules, they were ready to generalize their learning to a new context: dance. To introduce this last component, Katz broke into what the children initially thought was a spontaneous dance: He skipped twice, clapped once, and then paused. Next, he skipped four times, clapped once, and paused again. After completing the fourth position (eight skips, and one clap), Katz stopped and asked his students to explain what he had just done. They responded with giggles and looks of bewilderment. He danced the function again. This time he counted the skips and claps as he performed each movement. Hands began to shoot up, and excitement spread through the classroom. Students could now see that their teacher was not only dancing but also performing a times-two-plus-one pattern ($y = 2x + 1$). After several more teacher-led examples, Katz invited students to create their own dances, either alone or with partners.

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Unlike the pattern-building and function-machine activities, in which students had progressed from one-step to two-step patterns, we modeled only two-step patterns in the dance, and the children naturally composed movements that were based on two-step rules. This was true for struggling students as well as for the more advanced. The range of complexity in chosen numbers varied just as much as the numbers had during the previous components. For example, Sofia, a lower-achieving student, chose to dance $y = 5x + 3$, reasoning that she was comfortable with counting by fives. Other students challenged themselves, choosing to work with four, six, or nine as their coefficients. Naturally, students were creative in their choice of movements: crawling and rolling as well as performing twirls, hops, and jumps. Katz challenged students further, suggesting that they experiment with different elements of dance, such as levels (high vs. low), directions (side to side), and sharpness of movement.

Moving about the room, the teacher offered support with calculations or by assuming an audience role and guessing and explaining the dancers' functional rules. After much practice, during which students' calculations became more fluid and their dance movements more refined, Katz invited everyone to show the dances to the class. This exercise helped students further revise their dances before their final performance. As the opening vignette demonstrates, the final performance was met with class-wide enthusiasm. Furthermore, what was particularly noteworthy was the discovery that every child, even those who had previously struggled with two-step patterns, was able to create and recognize elements of two-step dance patterns and accurately perform their arithmetic calculations.

Clearly, the overall success of the dance component was due to the many and varied experiences the students had learned from activities based on geometric patterns and function rules. Reflecting on why even struggling students experienced such success with two-step dance routines, we can only speculate. First, it seemed to us that students were better able to keep track of counts and perform the proper arithmetic calculations with the accompaniment of rhythmic movement. Just as fingers can be used to help with math, so too bodies can help during algebraic dance. Second, when learning two-step functions, students sometimes struggle to differentiate between the coefficient and the constant. Creating and performing algebraic dance, much like building geometric patterns, helps to isolate these two components of an algebraic function.

Effective differentiation

Each component of the unit contributed to students' growth in algebraic understanding, and the effectiveness of the unit lay in the combination of all three components. Integrating visual, numeric, and kinaesthetic representations catered to different learning styles as well as provided multiple entry points for students of varying abilities. Furthermore, throughout the unit, students had opportunities to exercise creativity and free choice, necessary elements of effective differentiated instruction (Small 2009). This multifaceted patterning and algebra unit can be adapted and taught across different grades and populations.

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