

Place Value: Teaching for Understanding

Tiferet Nashman and Hilary St-Pierre

Essential Ideas of Place Value:

- Our number system is based on grouping by ten ($1 \times 10 = 10$, $10 \times 10 = 100$, etc.)
- The position of a digit determines its value or quantity (the 3 in $342 = 300$)
- Our way of describing and recording multi digit numbers is a cultural convention and non-obvious

How Place Value Understanding Develops:

- A schema of the place value system is developed before starting school (Mix, Prather, Smith, & Sockton 20..).
- The way place value is commonly taught fails to build on prior knowledge, ignores intuitive understanding, and doesn't equip students with a deep understanding of place value (Graven, Venkat, Westaway, & Tshesane 2013)
- Early understanding of place value is a determinant of future math achievement (Chan, Au, & Tang 2013)

Students with poor understanding of place value...

- Confuse cardinality (total number of objects) with unitizing (grouping by ten) (McGuire and Kinzie 2013)
- Do not work flexibly with numbers (Graven, Venkat, Westaway, & Tshesane 2013)
- Lose sense of number meaning and quantity (Graven, Venkat, Westaway, & Tshesane 2013)
- Struggle to read or record numbers (Chan, Au, & Tang 2013)
- Make order-of-magnitude errors (Chan, Au, & Tang 2013)
- Struggle with base-ten operations, such as carrying over and borrowing (Chan, Au, & Tang 2013)

Resources:

- *Teaching Arithmetic* by Maryann Wickett and Marilyn Burns
- The Game of Tens and Ones: https://store.mathsolutions.com/pub/media/documents/doc/0-941355-45-4_L.pdf
- Math Stories: <https://www.stem.org.uk/resources/elibrary/resource/30754/teaching-place-value>
- Looking at Number and Place Value, Marian Small:
<http://www.onetwoinfinity.ca/presentations/SingaporeMathGroup.pdf>

References:

- Bartolini Bussi, M. G. (2011). Artefacts and Utilization Schemes in Mathematics Teacher Education: Place Value in Early Childhood Education. *Journal of Math Teacher Education*, 93-112.
- Byrge, L., & Smith, L. B. (2014). Beginnings of Place Value: How Preschoolers Write Three-Digit Numbers. *Child Development*, 82(2), 437-443.
- Foster, C. D. (2012). Field Day. *Teaching Children Mathematics*, 18(8), 464-467.
- Fraivillig, J. L. (2017). Enhancing Established Counting Routines to Promote Place-Value Understanding: An Empirical Study in Early Elementary Classrooms. *Early Childhood Education Journal*, 21-30.
- Hiebert, J., & Wearne, D. (1992). Links Between Teaching and Learning Place Value with Understanding in First Grade. *Journal for Research in Mathematics Education*, 98-122.
- Magargee, S. D., & Beauford, J. E. (2016). Do Explicit Number Names Accelerate Pre-Kindergarteners' Numeracy and Place Value Acquisition. *Educational Studies in Math*, (92), 179-192.
- McGuire, P., & Kinzie, M. B. (2013). Analysis of Place Value Instruction and Development in Pre-Kindergarten Mathematics. *Early Childhood Education Journal*, (41), 355-364.
- Tanase, M. (2013). The Impact of Interactive Factors on Romanian Students' Understanding of Place Value. *SAGE Open*, 1-17.

Early Years: students naturally use “expansion” (Byrge and Smith 2014)

Instruction: Creating Connections

1. Intuitive expanded notation: “Forty-two” written as 402
2. Separate numbers: “Forty two” written as [40][2]
3. Use manipulatives: *Take these 42 cubes and put them in towers of ten. How many towers do we have? How many cubes don’t fit in our towers?*
4. Consolidate values: [40] [2] = 4t’s and 2 ones
5. Math notation: *We have 42 ones, so we have 4 groups of ten ones, and 2 extra ones. Record the number of groups of ten ones and the number of extra ones: 42*

Assessment: Place Value Interview

(Marilyn Burns)

1. On a sheet of paper, put out 16 cubes. “Here are 16 cubes.”
2. Ask the student to write the number 16 (large)
3. Gather 6 of the cubes next to the 6 in the 16. “These cubes show what the “6” in the number 16 means.”
4. Ask the student: “Show with the cubes what the “1” in the number 16 means

Primary: Place value decomposition (Graven, Venkat, Westaway, & Tshesane 2013)

Instruction: Expanded Addition Without Regrouping

Math notation: $134 + 123$

Expanded notation: [100] [30] [4] + [100] [20] [3]

Expanded notation as addition: [100 + 30 + 4] + [100 + 20 + 3]

Group like terms: $100 + 100 + 30 + 20 + 4 + 3$

Add like terms: $200 + 50 + 7$

Combine for final answer: 257

Instruction: Expanded Addition With Regrouping

Math notation: $194 + 128$

Expanded notation: [100] [90] [4] + [100] [20] [8]

Expanded notation as addition: [100 + 90 + 4] + [100 + 20 + 8]

Group like terms: $100 + 100 + 90 + 20 + 4 + 8$

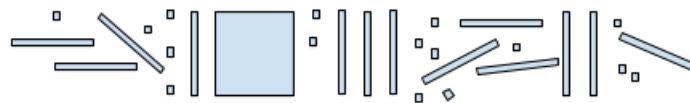
Add like terms: $200 + 110 + 12$

Re-expand: $200 + [100 + 10] + [10 + 2]$

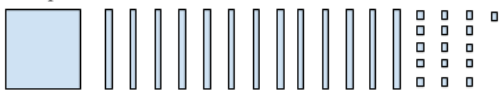
Add like terms: $300 + 20 + 2$

Combine for final answer: 322

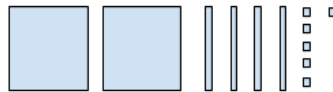
Assessment: Strategic Counting (Chan, Au, & Tang 2013)



1. Group:



2. Recombine:



3. Add:

$$200 + 40 + 6 = 246$$

Junior: Place value through operations (Castello 2016)

Instruction: Flexible Problem Solving

Math notation:

$$24 \times 16$$

Expanded notation:

$$[20 + 4] \times [10 + 6]$$

Grid notation:

	20	4
10	200	40
6	120	24

Long multiplication:

$$\begin{array}{r} 24 \\ \times 16 \\ \hline \end{array}$$

*no meaningless statements: zero is not a “place holder”

*students can judge the reasonableness of their responses based on place value

Examples of Open-Ended Place Value Questions (Marian Small)

A number includes the word twenty and the word three when you read it. What could it be?

A number has to be increased to round to the nearest 10 but decreased to round to the nearest 100. What could it be?

Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count.

I use exactly 12 base-ten blocks to represent a number. What might it be?

A number with a lot of 9s is less than a number with a LOT of 1s. How is that possible?