Place Value: Teaching for Understanding
Tiferet Nashman and Hilary St-Pierre

Essential Ideas of Place Value:
• Our number system is based on grouping by ten (1 x 10 = 10, 10 x 10 = 100, etc.)
• The position of a digit determines its value or quantity (the 3 in 342 = 300)
• Our way of describing and recording multi digit numbers is a cultural convention and non-obvious

How Place Value Understanding Develops:
• A schema of the place value system is developed before starting school (Mix, Prather, Smith, & Sockton 20..).
• The way place value is commonly taught fails to build on prior knowledge, ignores intuitive understanding, and doesn’t equip students with a deep understanding of place value (Graven, Venkat, Westaway, & Tshesane 2013)
• Early understanding of place value is a determinant of future math achievement (Chan, Au, & Tang 2013)

Students with poor understanding of place value...
• Confuse cardinality (total number of objects) with unitizing (grouping by ten) (McGuire and Kinzie 2013)
• Do not work flexibly with numbers (Graven, Venkat, Westaway, & Tshesane 2013)
• Lose sense of number meaning and quantity (Graven, Venkat, Westaway, & Tshesane 2013)
• Struggle to read or record numbers (Chan, Au, & Tang 2013)
• Make order-of-magnitude errors (Chan, Au, & Tang 2013)
• Struggle with base-ten operations, such as carrying over and borrowing (Chan, Au, & Tang 2013)

Resources:
• Teaching Arithmetic by Maryann Wickett and Marilyn Burns
• The Game of Tens and Ones: https://store.mathsolutions.com/pub/media/documents/doc/0-941355-45-4_L.pdf
• Math Stories: https://www.stem.org.uk/resources/elibrary/resource/30754/teaching-place-value
• Looking at Number and Place Value, Marian Small: http://www.onetwoinfinity.ca/presentations/SingaporeMathGroup.pdf

References:
**Early Years:** students naturally use “expansion” (Byrge and Smith 2014)

- **Instruction: Creating Connections**
  1. Intuitive expanded notation: “Forty-two” written as 402
  2. Separate numbers: “Forty two” written as [40][2]
  3. Use manipulatives: Take these 42 cubes and put them in towers of ten. How many towers do we have? How many cubes don’t fit in our towers?
  4. Consolidate values: [40] [2] = 4t’s and 2 ones
  5. Math notation: We have 42 ones, so we have 4 groups of ten ones and 2 extra ones. Record the number of groups of ten ones and the number of extra ones: 42

- **Assessment: Place Value Interview** (Marilyn Burns)
  1. On a sheet of paper, put out 16 cubes. “Here are 16 cubes.”
  2. Ask the student to write the number 16 (large)
  3. Gather 6 of the cubes next to the 6 in the 16. “These cubes show what the “6” in the number 16 means.”
  4. Ask the student: “Show with the cubes what the “1” in the number 16 means

---

**Primary:** Place value decomposition (Graven, Venkat, Westaway, & Tshesane 2013)

- **Instruction: Expanded Addition Without Regrouping**
  Math notation: \(134 + 123\)
  Expanded notation: \([100][30][4] + [100][20][3]\)
  Expanded notation as addition: \([100 + 30 + 4] + [100 + 20 + 3]\)
  Group like terms: \(100 + 100 + 30 + 20 + 4 + 3\)
  Add like terms: \(200 + 50 + 7\)
  Combine for final answer: 257

- **Instruction: Expanded Addition With Regrouping**
  Math notation: \(194 + 128\)
  Expanded notation: \([100][90][4] + [100][20][8]\)
  Expanded notation as addition: \([100 + 90 + 4] + [100 + 20 + 8]\)
  Group like terms: \(100 + 100 + 90 + 20 + 4 + 8\)
  Add like terms: \(200 + 110 + 12\)
  Re-expand: \(200 + [100 + 10] + [10 + 2]\)
  Add like terms: \(300 + 20 + 2\)
  Combine for final answer: 322

- **Assessment: Strategic Counting** (Chan, Au, & Tang 2013)
  1. Group:
  2. Recombine:
  3. Add:

```
  200 + 40 + 6 = 246
```

---

**Junior:** Place value through operations (Castello 2016)

- **Instruction: Flexible Problem Solving**
  Math notation: \(24 \times 16\)
  Expanded notation: \([20 + 4] \times [10 + 6]\)
  Grid notation: 
  Long multiplication: 

```
  24
  x 16
  240
  240
  144
  | 384
  | 480
  24
```

*no meaningless statements: zero is not a “place holder”

*students can judge the reasonableness of their responses based on place value

---

**Examples of Open-Ended Place Value Questions** (Marian Small)

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number includes the word twenty and the word three when you read it.</td>
<td>What could it be?</td>
</tr>
<tr>
<td>A number has to be increased to round to the nearest 10 but decreased</td>
<td>What could it be?</td>
</tr>
<tr>
<td>A number with a lot of 9s is less than a number with a LOT of 1s.</td>
<td>How is that possible?</td>
</tr>
<tr>
<td>Given a two-digit number, mentally find 10 more or 10 less than the</td>
<td>What might it be?</td>
</tr>
<tr>
<td>I use exactly 12 base-ten blocks to represent a number.</td>
<td></td>
</tr>
<tr>
<td>A number has to be increased to round to the nearest 10 but decreased</td>
<td></td>
</tr>
</tbody>
</table>