3

Developing spatial reasoning

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In brief …

Once believed to be a fixed trait, there is now widespread evidence that spatial reasoning is malleable and can be improved in people of all ages. In this chapter, we first discuss the relationship between spatial reasoning and mathematics and then we present a variety of spatial training approaches that have been shown to be effective not only in supporting children's spatial reasoning but also mathematics performance.

Spatial reasoning as a foundation for mathematics learning

There is an emerging consensus that spatial reasoning plays a foundational role in the early development of mathematics. Due in part to the recent design of age-appropriate measures of spatial reasoning for young children (see Chapter 2, Textbox 1), researchers have begun to understand how early spatial skills relate and contribute to the learning of school mathematics. In a longitudinal study, that followed children from the ages of 3 to 5, Farmer et al. (2013) found evidence to suggest that children's spatial skills at 3 years of age were strong predictors of how well the same children performed in mathematics two years later, upon formal school entry. Moreover, spatial skills were better predictors of later mathematics performance than vocabulary and even mathematics.

In another study, Verdine and et al. (2014) reached a similar conclusion. Spatial skills assessed at the age of 3, along with executive function skills assessed at the age of 4, predicted over 70% of the variability in mathematics performance at 4 years of age. Even after controlling for the contribution of executive functions, spatial skills predicted 27% of the variability in children's mathematics performance. It is worth noting that in both of the above studies, the researchers used a relatively simple means to assess children's spatial reasoning. Children were presented with Mega-Block™ arrangements and asked to copy them as accurately as possible (see Figure 3.1). A score was assigned to each child based on how accurately he or she was able to replicate the design.

Figure 3.1: Stimuli for the Test of Spatial Assembly (TOSA; see Verdine et al., 2014b).

We mention how the researchers assessed spatial skills because we think it is relevant for the consideration of early classroom interventions. Indeed, there is sufficient evidence suggesting that early construction skills – of the sort that involve copying, drawing, and block building – play an important role in the learning of mathematics (Casey, Andrews et al., 2008; Casey, Erkut, Ceder, & Young, 2008; Tzuriel & Egozi, 2010). For example, Wolfgang et al. (2001) carried out a longitudinal study that followed children from preschool to adulthood (a period spanning 16 years). The researchers showed the complexity of block building at age 5 was a significant predictor of how well the same individuals performed in high school mathematics.

Taken together, the above research findings suggest that spatial reasoning and mathematics are co-related and that early spatial skills may provide a foundation on which mathematics learning is built. This raises the question of how and why mathematics and spatial reasoning are related.

How and why spatial reasoning helps "do" mathematics

The question of how and why spatial reasoning and mathematics are linked remains largely unknown. In their recent review on the subject, Mix and Cheng (2012) urged the field to move beyond correlational studies, stating:

> The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related. Rather, we need to know why the two are connected – the causal mechanisms and shared processes – for this relation to be fully leveraged by educators and clinicians. (p. 206)

The last part of their statement is a particularly important point. In order to fully harness and develop the powers of spatial reasoning in our mathematics classrooms, we need to have a strong theoretical stance and evidence-based knowledge as to why the two go hand in hand. As mathematicians, mathematics educators, teachers and curriculum developers, we need to work together to understand more about the connections between spatial reasoning and mathematics. Indeed, teachers need to be able to recognize and theorize when spatial skills are needed to support mathematics learning, as well as when a focus on number might hinder or prevent mathematical understanding (Newcombe, 2014; Whiteley, 2014).

Perhaps an example better serves this point. A teacher would have little difficulty explaining to a curious parent why so much time was being spent on developing number sense. The practicality is self-evident; numbers are ubiquitous and endlessly useful and for this reason, it is unlikely that a parent would even ask such a rhetorical question. However, if asked to explain why so much time was being spent (in math class nonetheless!) on developing spatial reasoning, a teacher would likely be facing a much more difficult challenge. The relationship between spatial reasoning and mathematics is not always immediately apparent, and yet, decades of research inform us that the two are intimately connected. Based on prior research, and our own experiences working in classrooms, we offer three reasons why spatial reasoning is related to and helps support the learning of mathematics.

Mathematics is inherently spatial

To "do," "create," and "express" mathematics is to use and depend on spatial reasoning and spatial representations. As mentioned in Stanislas Dehaene's book, *The Number Sense* (2011), it is "almost as if they (spatial reasoning and mathematics) were one and the same ability" (p. 135). Clements and Sarama (2011) posit that it is through mathematics that we "communicate ideas that are essentially spatial. From number lines to arrays, even quantitative, numerical, and arithmetical ideas rest on a geometry base" (p. 134). Indeed, in our own work in early years classrooms, we are regularly confronted with examples of how spatial reasoning and mathematics are intimately linked. Linear and area measurement, early patterning and algebra, fractions, symmetry, and not to mention geometry, are inextricably linked to children's understandings of spatial relationships. Even something as simple as comparing shapes or numbers becomes an act of spatial reasoning when the objects assume different orientations. Interestingly, research suggests that the role of spatial reasoning and the use of spatial representations become even more important as one advances in their learning of mathematics (Mix & Cheng, 2012). In the following quote we are reminded to continually pay attention to the highly visual and spatial nature of calculus.

> The role of visual thinking is so fundamental to the understanding of calculus that is difficult to imagine a successful calculus course which does not emphasize the visual elements of the subject. This is especially true if the course is intended to stress conceptual understanding, which is widely recognized to be lacking in many calculus courses as now taught. Symbol manipulation has been overemphasized and in the process the spirit of calculus has been lost. (Zimmermann, 1991, p. 136)

It is easy to lose sight of the importance of spatial reasoning in mathematics. The representations used in spatial reasoning are often private or internal to the individual learner, and as such, are often difficult to externalize and share through external community conventions, or rather lack thereof (Whiteley, 2014). In many ways, spatial reasoning is so much a part of mathematics that we take it for granted, we forget to acknowledge its role, and we do little to harness its potential (see Clements & Sarama, 2004).

Numbers are represented spatially

For over a century now, researchers have revealed a close relationship between space and numbers (Galton, 1880; Mix & Cheng, 2012). Dating back to the late 1800s, Sir Francis Galton provided anecdotal evidence that for some, individual numbers were seen in the "mind's eye" as objects that occupied distinct visual and spatial forms:

> Those who are able to visualize a numeral with a distinctness comparable to reality, and to behold it as if it were before their eyes, and not in some sort of dreamland, will define the direction in which it seems to lie, and the distance at which it appears to be. If they were looking at a ship on the horizon at the moment that the figure 6 happened to present itself to their minds, they could say whether the image lay to the left or right of the ship, and whether it was above or below the line of the horizon; they could always point to a definite spot in space, and say with more or less precision that that was the direction in which the image of the figure they were thinking of first appeared. (1881, p. 86)

These number forms as Galton referred to them, provided one of the earliest accounts of a suspected link between numerical and visual-spatial processes. Galton noted that the experience of number forms was a relatively stable trait within individuals, but large variation existed between individuals. The visual-spatial properties associated with number forms varied according to spatial orientation, color, brightness, and perceived weight (see Figure 3.2; Galton, 1880; Galton, 1881). Taken together, this work suggested that numbers were internally represented as objects and occupants of distinct positions in linear space (Galton, 1880; de Hevia, Vallar, & Girelli, 2008).

During the last several decades, there has been resurgence in the scientific study of how humans mentally represent numbers (c.f. Dehaene, Bossini, & Giraux, 1993). There is now extensive support for Galton's intuitions about the visual-spatial nature of numerical representations (de Hevia et al., 2008; Seron, Pesenti, Noël, Deloche, & Cornet, 1992). While only a small segment of the population (approximately 12%) experience the vivid number forms described by Galton, the vast majority unconsciously represents numbers spatially (Sagiv, Simner, Collins, Butterworth, & Ward, 2006). For example, numerous studies show an automatic association of small numbers as belonging to the left side of space and larger numbers as belonging to the right side of space, a finding referred to as the SNARC effect (Spatial-Numerical Associations of Response

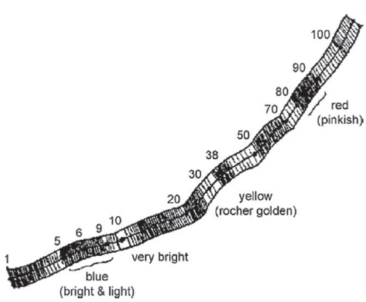


Figure 3.2: A number form described by one of Galton's subjects (from Galton, 1880).

Codes; Dehaene, 1993). As such, people are faster to respond to smaller numbers (e.g., 1, 2, 3) with their left hand and faster to respond to numbers of larger magnitude with their right hand (e.g., 8, 9, 10). The reverse is true in societies that write and read numbers from a right-to-left orientation, such as the case in Palestine (e.g., see Shaki, Fischer, & Petrusic, 200).

The influences of spatial representations of number are also present during simple arithmetic (Fischer & Shaki, 2014). In a recent special issue on the topic, a collection of findings demonstrated that not only are single digits subject to spatial biases, but arithmetic and even the operators themselves (i.e., plus (+) and subtraction (–) symbols) are associated with space (Fisher & Shaki, 2014). Addition problems elicit "right-side-of- space" biases whereas subtraction problems elicit "left-side-of-space" biases. Evidence of such spatial biases can be seen in the tendency for people to overestimate the result of addition problems and underestimate the result of subtraction problems, an effect referred to as the operational momentum (OM) effect (Fischer & Shaki, 2014). Werner and Raab (2014) discovered a link between the direction of people's eye movements and type of problem they were solving. The authors' revealed a shift in attention toward the right side of space for addition solutions and a shift in attention toward left side of space for subtraction solutions. Together, these and other findings (see Fischer & Shaki, 2014), are thought to reflect the cognitive representation of magnitude meaning along a metaphorical "mental number line."

Other examples of how the "mental number line" might be implicated during mathematics come from further studies from the psychology literature that utilize actual number lines. Over the past decade, there has been an explosion of research on the use and implications of findings related to number line estimation tasks. In a typical number line task, participants are presented with a line with only two end points (e.g. 0–10, 0–100, 0–1000; see Figure 3.3). Participants are then presented with a number and asked to indicate its exact location on the line. Performance on the task is thought to reflect the precision of an individual's mental number line or mental counting line. Importantly, performance on the task has been found to strongly predict concurrent and later mathematics performance (Siegler & Booth, 2004; Booth & Siegler, 2006). The findings of Siegler and others is that with age, experience, and training, children's number line performance improves as a function of more accurate mappings of numbers to space.

Further evidence of a link between a spatial representation of number and arithmetic performance comes from Gunderson et al. (2012). In a longitudinal study involving two data sets, Gunderson et al. (2012) found that spatial skills (i.e., mental transformation skills involving rotation and translation) at the

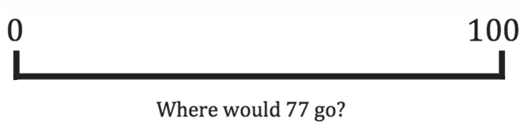


Figure 3.3: Example item from a number line estimation task.

beginning of 1st and 2nd grades predicted growth in linear number knowledge (as assessed with a number line estimation task) over the course of the school year. Furthermore, children's spatial skills at age five predicted how well these same children performed three years later on an approximate symbolic calculation task. Interestingly, this relationship was mediated by children's number line performance during 1st grade. This finding is significant in that it suggests that spatial skills play an important role in the development of young children's spatial representations of number. Taken together, this line of research suggests that mathematics and spatial reasoning are tied together through the spatial representation of numbers. Preliminary research attempting to leverage this connection to improve number sense performance is encouraging. Research has shown that playing linear number board games for even one hour can increase at-risk preschool students' abilities to make number line estimations, and judge and compare numerical magnitudes (Siegler & Ramani, 2008; Siegler & Ramani, 2009).

This human tendency to represent numbers spatially is further supported by research in neuroscience. It is now widely recognized that both numerical and visual-spatial tasks require and depend on the recruitment of highly similar brain regions, namely, various neural networks located within the parietal cortex (de Hevia et al., 2008; Hubbard, Piazza, Pinel, & Dehaene, 2009b). For example, the spatial task that involves mental rotation and mathematical tasks requiring numerical processing, are both thought to rely on the intraparietal sulcus located within the parietal lobe (Hubbard et al., 2009b; Hubbard, Piazza, Pinel, & Dehaene, 2009a). Indeed, Hubbard et al. (2009) suggest that, "…the parietal mechanisms that are thought to support spatial transformation might be ideally suited to support arithmetic transformations [e.g., calculations] as well" (p. 238).

The above evidence suggests that spatial and numerical processes are closely linked and that space is a useful metaphor for how we think about numbers (Hubbard et al., 2009a; 2009b). However, more research is needed to reveal the specific mechanisms that underlie this relationship, and furthermore, to elucidate how spatial representations of number are related to mathematics more generally, including areas of mathematics that extend beyond simple arithmetic and calculations.

Mathematics and spatial reasoning involve visual-spatial working memory

Another way that spatial reasoning and mathematics might be linked is through shared cognitive resources, including the ability to mentally manipulate visual-spatial information. Visual-spatial working memory appears to be especially important for the early learning of mathematics. Children who have an easier time remembering and mentally manipulating visual-spatial information tend to have an easier time doing mathematics (Kyttälä & Lehto, 2008; Kyttälä, Aunio, Lehto, Van Luit, & Hautamaki, 2003; Thompson, Nuerk, Moeller, & Cohen Kadosh, 2013). Given this basic relationship, some researchers have found evidence (albeit somewhat controversially) that training-induced improvements in working memory (e.g., through computerized training exercises) results in improved mathematics performance (Holmes, Gathercole, & Dunning, 2009; St. Clair-Thompson, Stevens, Hunt, & Bolder, 2010; Witt, 2011). For the time being, these findings suggest a cause-effect relationship whereby improvements in working memory can be expected to aid in the performance of mathematics tasks.

Recall that it is intrinsic-dynamic spatial reasoning that has been, to date, most associated with performance in mathematics. Mix and Cheng (2012) hypothesized that the strength of this relationship depends on the shared demands placed on visual-spatial working memory. That is, both mathematics and intrinsic-dynamic spatial reasoning require the active maintenance and manipulation of visual-spatial information in one's mind. Therefore, it is possible that classroom interventions aimed at developing intrinsic-dynamic spatial reasoning might also strengthen children's visual-spatial working memory capacity – a core cognitive skill involved in the learning of mathematics. Future research efforts are needed to test this possibility along with a more detailed account of the role of visual-spatial working memory in spatial reasoning.

In returning to the question of why spatial reasoning matters for mathematics, we have presented evidence to suggest that 1) many mathematics tasks use inherently spatial representations (e.g., linear and area measurement, visualizing multiplication as arrays, geometrical transformations), 2) numbers are commonly represented spatially, and 3) both mathematics and spatial reasoning rely visual-spatial working memory. In the next section, we examine the malleability of spatial reasoning and what this means for mathematics learning, and consider how these three accounts might be utilized in the design of effective classroom interventions that aim to bridge the mathematics and space divide.

Spatial reasoning can be improved through practice and targeted interventions

The strong link between spatial reasoning and mathematics raises the possibility that improving children's spatial skills might serve as a way to strengthen mathematics learning. To date, however, very few studies have pursued this line of inquiry. Uncertainty about the malleability of spatial reasoning may be one reason for this. After all, the possibility of improving math performance through spatial learning depends to a large extent on whether spatial skills can be taught and learned.

Historically, spatial ability has been viewed as a core aspect of intelligence. Beginning in the early 20th century and spanning to the present day, psychologists have consistently identified spatial ability as an essential factor in the study and definition of intelligence. Perhaps owing to the close tie between spatial ability and intelligence, spatial reasoning is commonly viewed as a fixed intellectual trait – "either you have it or you don't" (Newcombe, 2010). It is not uncommon, for example, to hear someone remark that they "don't read maps," "can't follow directions" or even go so far saying they "have no spatial sense whatsoever." This "fixed" viewpoint appears to be based on misconceptions and false belief.

Decades of research confirm that spatial reasoning is malleable and subject to improvement with practice and targeted interventions. The most conclusive evidence that spatial reasoning is malleable comes from a recent meta-analysis that analyzed 206 spatial training studies over a 25-year period (1984-2009; Uttal et al., 2013). The study concluded that people of all ages and through a wide assortment of spatial training interventions (e.g., video games, course training, spatial task training) demonstrated significant gains in spatial reasoning. Moreover, the average effect size of training was large and approached a half standard deviation (0.47). To put this effect in context, an improvement of this magnitude would approximately double the number of people with the spatial skills associated with being an engineer (see Uttal et al., 2013). Indeed, the implications of improving spatial skills are significant and far reaching, especially in relation to the ever-important STEM disciplines. Verdine et al. (2014) go so far as to suggest that spatial instruction can be expected to have a "two-for-one" effect, yielding benefits in both spatial reasoning and mathematics.

Although the majority of studies (67%) in the meta-analysis measured spatial skills immediately after training, some studies demonstrated that the effects of training persisted over time. In one longitudinal study, the training effects were still present four months after the intervention (Feng, Spence, & Pratt, 2007). Another notable feature of the meta-analysis was the finding of nearly identical near and far transfer effects. That is, training of one spatial skill led to improvements on spatial tasks closely related to the trained skill (i.e., near transfer) as well as spatial tasks that were quite distinct from the trained skill (i.e., far transfer). For example, in two studies, mental rotation training resulted in improved mental rotation skills (i.e., near transfer), but also led to more generalized mental transformation skills, as evidenced by improvements on a mental paper folding test (Wright et al., 2008; Chu & Kita, 2011).

Overall, the results of the meta-analysis performed by Uttal et al. (2013) go against the common misconception that spatial reasoning is "fixed" and consequently "unteachable." On the contrary, spatial reasoning appears to be highly malleable. Moreover, a wide variety of training methods appear effective in bringing about durable and transferable improvements in people of all ages.

A survey of interventions and activities to support young children's spatial reasoning

In this section, we provide an overview of the types of interventions and activities that have been found to support the development of young children's spatial reasoning, including block building, puzzle play, drawing exercises, and paper-folding activities, including origami. Each one of these activities simultaneously targets a number of important spatial skills and to varying extents all encourage the development of spatial visualization skills – a feature and type of spatial reasoning that is closely linked to mathematics performance (Mix & Cheng, 2012). Together, the interventions detailed below offer an assortment of "easy-to-implement" classroom activities and ideas for lessons that provide multiple entry points to engage, support, and improve students' spatial reasoning skills.

Construction lay

Construction play with materials, such as wooden blocks, Lego™, and Meccano™ toys, has been closely linked with the development of spatial reasoning (e.g., Casey et al., 2008; Nath & Szücs, 2014). Construction play affords opportunities to develop spatial reasoning through physical and visual experiences involving the composition and decomposition of 3D structures, perspective taking (e.g., moving around one's structure), symmetry, and transformations (e.g., rotations, translations, reflections). Although interest and time spent engaging in construction play has been related to spatial reasoning (c.f. Doyle, Voyer, & Cherney, 2012; Robert & Héroux, 2003), more recent research has revealed that it is the quality, both accuracy and complexity, of the building that seems most salient.

Casey et al. (2008), for example, conducted an intervention study with kindergarten children in which they studied the effectiveness of different types of block-play on students' spatial reasoning skills. In this study kindergarten classrooms were assigned to one of three block-building groups. One of the groups engaged in free, unguided block play. A second group of students were given specific building goals for their block play (e.g., build a wall that could be used to contain animals). Finally, the third group were provided with these same building goals but embedded in a narrative (e.g., a story involving a dragon, Sneeze, who required help re-building a series of fallen down castles). Importantly, in all three conditions, children spent an equal amount of time engaging in block play. Both before and after the various interventions, all of the children were assessed on a number of measures including an assessment of block building complexity, 3D mental rotation, and block design – a common intelligence test that involves recreating 2D geometry designs using variously coloured and patterned cubes.

Results showed that, compared to those who engaged in free block play, children in the goal-directed block play groups demonstrated significant gains on the block design test. However, only those children in the narrative condition demonstrated significant gains in their block building performance. These findings are important as they indicate that the quality of block building influences the development of children's spatial reasoning skills. This study adds to a growing body of research that demonstrates the importance of providing children with teacher-guided block-play that involves specific building goals (e.g., see Reifel & Greenfield, 1982; Gregory, Kim & Whiren, 2003; Hanline, 2001).

Puzzle play

There is growing recognition that puzzles (e.g., jigsaw, Tangrams™, pentomino challenges, etc.) provide a meaningful opportunity, especially early in development, to build spatial skills. Just like construction play, puzzles target a number of different spatial skills, including composition and decomposition of shapes, mental rotation, and spatial reasoning (e.g., "this one must be a corner piece"). One recent study demonstrated just how important early puzzle play might be in contributing to the development of spatial skills (Levine, Ratcliff, Huttenlocher, & Cannon, 2012). In this study, the authors observed child-parent(s) interactions in their homes every four months while the child was between 2 and 4 years. When the children were 4.5 years, they were assessed on a spatial task that involved mental transformations of various 2D shapes (see CMTT in Table 2.1). This study yielded two important findings: children who were observed playing with puzzles during the visits over the 2 years performed better on the spatial task than those who did not engage in puzzle play. This relationship held even after controlling for important parental variables, such as, education, household income, and parental language. Moreover, the frequency and quality of puzzle play – amongst those who did play with puzzles – was further predictive of how well the children performed on the task. This study suggest that even before formal school entry certain home activities, such as puzzle play, are related to later spatial skills (Doyle, Voyer, & Cherney, 2012; Robert & Héroux, 2003).

Cross-sectional studies conducted with early elementary students have further solidified a link between puzzle performance and spatial reasoning skills. For example, Verdine et al. (2008) found high correlations between performance on a standard jigsaw puzzle and measures of mental rotation, spatial perception, and spatial visualization. In an attempt to harness the relationship between puzzle play and certain geometry and spatial skills, one group of researchers designed and carried out a one-month "puzzle" intervention (Casey, Erkut, Cedar, & Young, 2008). Children were either assigned to a control condition (i.e., free play) or an experimental condition that involved listening and responding to a narrative. As part of the experimental condition, students worked through a series of open-ended puzzle tasks involving Tangrams. The results indicated that all boys – regardless of group assignment – demonstrated approximately equal gains on a pre- and post-test involving various puzzle tasks (designed to assess part-whole understanding). Interestingly, girls in the experimental condition but not the control group demonstrated significant gains on the task. This study suggests that, at least for young girls, puzzle play might be one effective approach for improving the early understanding of part-whole relations.

Perhaps the most promising puzzle intervention to date involves training with the video game Tetris (Okagaki & Frensch; Terlecki, Newcombe, & Little, 2008); a fast paced puzzle game that involves rotating and translating polyominoe shapes into the most optimal position. Although studies have only been carried out with adolescents and adults, the findings from these studies suggest that game play results in improved mental rotation skills and spatial visualization. There is no reason to suspect that games such as Tetris would not also be useful in early learning settings.

Drawing tasks

At some point in elementary mathematics, one must confront the often-difficult task of interpreting and creating isometric drawings – a feat that requires spatial reasoning skills. In a number of studies with engineering students, researchers have demonstrated the promising effects of training these students in perspective drawing skills. Indeed very promising research studies have shown that intensive drawing practice, of the sort that involves learning how to accurately represent 2D and 3D objects, can significantly improve engineering students' spatial skills (Baartmans & Sorby, 1996; McAuliffe, 2003; Sorby, 2009).

Recent evidence suggests that drawing activities might also be an effective way of improving young children's spatial reasoning. For example, Tzuriel and Egozi (2010) carried out a drawing intervention with a population of first grade children. The intervention consisted of eight 45-minute sessions based on Quick Draw™ stimuli (see Wheatley, 1996). Working with small groups of children, the experimenter presented students with a 2D geometric design for only three seconds (see Figure 3.4 for an example). Children then had to draw the image from memory. This was followed by a group discussion that involved sharing how the images were initially perceived and remembered (e.g., "I saw an 'X' and a 'T' overlapping one another inside a square"). The experimenter facilitated the discussion and directed children to acknowledge the different perspectives amongst group members. Children were also encouraged to rotate the images and notice how different orientations influenced one's perspective. Compared to a control group, children who participated in the Quick Draw activities demonstrated significant improvements on two separate tests of mental rotation.

Paper folding

One of the most established tests of spatial visualization is called the Paper Folding Test (see Figure 3.5). In this test, participants are presented with a sequence of folds in a piece of paper. The folded piece of paper is then punctured with a hole punch. The objective is to determine what the piece of

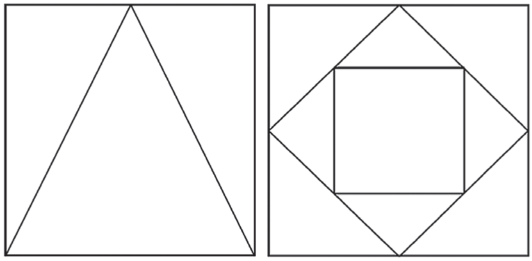


Figure 3.4: Example of the types of stimuli used in Quick Draw™. Students are presented with the image for three seconds and then must try to recreate the image from memory (for original Quick Draw™ images see Wheatley, 1996).

paper would look like if unfolded – how many holes would appear and where exactly would they be located? Given that spatial visualization lies at the heart of spatial reasoning, many interventions aim to improve this important skill. Thus an ideal candidate for improving spatial visualization involves interventions that utilize paper-folding tasks.

In one such study, Taylor and Hutton (2013) led a unit on origami and paper engineering with classrooms of fourth grade students. The intervention largely consisted of creating and deconstructing complex paper models and visualizing the results of making certain folds and cuts. Compared to a control group, children in the experimental group improved on two separate tests of spatial visualization, both of which involved mental paper folding. Furthermore, participants in the program reported high levels of engagement throughout the program.

Cakmak, Isiksal, & Koc (2014) also used origami as an instructional approach to teach geometrical and spatial reasoning skills. In this study, students in grades 4 through 6 participated in ten, 40-minute, in-class origami sessions. The teacher facilitated each session by first showing the students how to perform a certain fold and then had students follow. Throughout the instructional sequences students were encouraged to work together. After each folding stage, the class discussed the formed shapes and their properties (e.g., "Which shape do we have now?" "Why do you think we get this shape?" and "What are the properties of this shape?"). Upon completing the origami model, the teacher and students summarized the geometrical concepts and mathematical terms encountered throughout the model making. Not only did students demonstrate large gains on an extensive battery of geometry and spatial reasoning, but students also reported an increased awareness of how origami related to mathematics (e.g., geometrical transformations, fractions, 2D and 3D shapes, angles, etc.). For example, in the words of one student, "While making the samurai hat, we talked about the trapezoid, isosceles triangle, equilateral triangle, and scalene triangle. We also emphasized that the top and bottom bases of the trapezoid were parallel to each other. We folded the angles of 45° and 22.5°" (p. 65). Interestingly, other researchers have shown that paper folding offers a potentially powerful entry point into students' thinking about fractions and multiplicative reasoning (e.g., see Empson & Turner, 2006). Taken together, it appears as though paper folding is a useful tool for the teaching and learning of skills and concepts related to both spatial reasoning and mathematics.

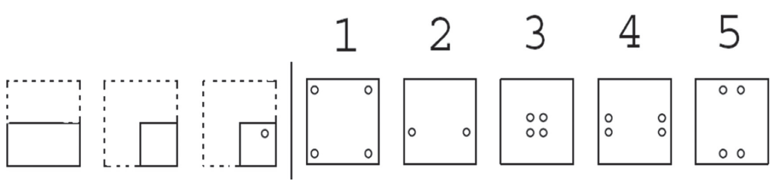


Figure 3.5: An example item from the Paper Folding Test (Chu & Kita, 2011).

Training spatial reasoning to support mathematics learning

Despite the historical relationship between spatial reasoning and mathematics, surprisingly few researchers have examined whether spatial training generalizes to mathematics learning. To our knowledge, there are only three studies that have directly tried to assess the use of spatial training for improvement in mathematics. In the next section, we review these three studies: one conducted in a laboratory setting; one in an afterschool program and one in early years classrooms.

Training on a spatial task

In the first study to causally demonstrate the effects of spatial training on mathematics, the researchers, Cheng and Mix (2013), assigned children to either a spatial training condition (i.e., mental rotation training) or crossword puzzle condition. Both before and after the intervention, children completed two spatial tasks and a test of mixed calculation problems. Children in the spatial training condition were trained on the Children's Mental Transformation Task (see Table 2.1). This training involved two steps. Children first were asked to visualize the solution to each problem. That is, to identify the correct solution amongst the four alternatives. Children then confirmed the accuracy of their response by putting together actual cardboard "cut outs" of the shapes. Thus, children were given immediate feedback about the accuracy of their mental transformations. In both conditions, the intervention lasted for a single 40-minute training session. Remarkably, children in the spatial training group, but not the crossword condition, demonstrated significant improvements not only on the mental transformation task – an expected finding – but also on the calculation test. Improvements were most evident on missing term problems (e.g., 5 + ___ = 7), a finding that was attributed to the possibility that training primed children to approach the problems through spatially reorganizing the problems (e.g., 5 + ___ = 7 becomes ___ = 7 – 5).

This is an important finding, as it the first empirical study to demonstrate the potential of spatial training as a means to facilitate calculation performance. However, caution is also warranted. Other studies are needed to replicate this finding. Not all brief interventions of this sort – no matter how well they are designed – will result in improved mathematics performance. Furthermore, failure to replicate findings from such a short intervention does not necessarily indicate that spatial training does not help facilitate mathematical understanding.

Indeed, it is our belief that while carefully controlled experimental studies are necessary for moving the field forward, we also need the types of training studies that take place in actual classrooms and for sustained periods of time. In addition, while the study above targeted only one specific spatial reasoning skill, little is known about how targeting multiple spatial skills might effect mathematics learning. What follows is a description of two studies that have attempted to address some of these issues.

A construction-based afterschool arts program

Working with underserved and at-risk preschool populations, Grissmer et al. (2013) designed and carried out an extremely intensive spatial intervention. Half the preschoolers were assigned to the experimental group and half were assigned to the "business as usual" control group. Children in experimental group took part in a seven-month intervention that was aimed at developing spatial and fine-motor skills. Four times a week, for a approximately 45 minutes, children took part in activities that involved creating and copying geometric designs made from a variety of materials, including Legos®, Wikki Stix®, and pattern blocks. Although both groups of children started at the same place in terms of their testing performance, there were marked differences between the two groups at the end of the intervention. Compared to the control group, those in the spatial intervention demonstrated gains in a number of areas, including spatial reasoning, self-regulation, and importantly in overall mathematics performance. In terms of mathematics performance, children in the spatial group advanced an impressive 17 percentile points, from 30th to 47th percentile, on a nationwide test of numeracy and problem solving. This finding provides some preliminary evidence that an intensive and sustained spatial program, that utilizes a number of different spatial tasks, is an effective means of supporting young children's mathematical development. This study also points to the promising effects of interventions that aim to strengthen young children's construction skills.

A seven-month in-class spatial reasoning intervention

As part of an ongoing professional development research project, Math for Young Children (e.g. Moss, Hawes, Naqvi, & Caswell, in press), researchers worked with a group of Junior Kindergarten to Second Grade teachers in three schools primarily serving First Nations populations. The researchers implemented an in-class intervention in which spatial reasoning tasks were incorporated into the regular mathematics curriculum (Moss, Hawes, Caswell, Naqvi, & MacKinnon, in preparation). Teachers in both the experimental and control groups participated in separate professional development sessions. The teachers from the experimental group received professional development on teaching and learning of spatial reasoning; and teachers in the control group worked on inquiry approaches to environmental science.

The spatial reasoning intervention was delivered by the teachers over 7 months and consisted mainly of a series of brief spatial tasks, which became known as "rug activities." The rug activities included drawing, building, copying, and visualization exercises (see Table 3.1) and targeted the development of the young students' intrinsic-dynamic spatial reasoning (Mix & Cheng, 2012). These activities were carried out with the full class during "circle time" or with small groups at teacher-guided math centers. There were significant variations in how much time individual teachers devoted to these activities. However, on average, students participated in the activities three times a week for a total of approximately 40 hours throughout the school year. To assess the efficacy of the intervention, all of the students (N=67) participated

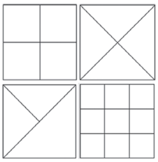
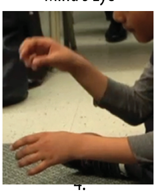
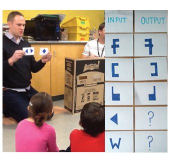
Name of "Rug Activity"	Description of Activity	Geometry and spatial skills targeted
1. Can you Draw this? 	• Children were provided with pieces of paper with an outline of a square on it • Children were then shown a geometric design composed within the square boundaries • After viewing the design for 10 seconds, children attempted to re-create (using a pencil) the exact design within the boundaries of their own square • Teachers facilitated discussions around strategies and different ways of remembering the designs • Note: this activity was based on Wheatley, 1996; also see Tzuriel & Egozi (2010) for a study on the effectiveness of this activity)	• Visual-spatial memory/ visualization • Composing/decomposing/ partitioning space • Proportional reasoning
2. Can you Build this? 	• Similar procedures to 'Can you Draw this?' • Children were shown a geometric structure composed of multi-link cubes • After viewing the structure for 10 seconds, children attempted to re-create the structure from memory using their own multi-link cubes • In another version of this activity, children were presented with a structure and asked to re-create it with no memory component	• Visual-spatial memory/visualization • Composing/decomposing 3D figures
3. Building with the Mind's Eye 	• Children were given oral instructions in how to build a 2D or 3D shape/figure (e.g., Take two blue cubes and attach them together, one on top of the other. Stand up the two attached cubes and make them look like a tower. Now take a red cube and attach to ... etc.) • Children built images of the shape/figure in mind, based on instructions given • After giving instructions, teacher showed children multiple shapes/figures and had children discuss/reason which one perfectly matched the description	• Visualization • Composition of 2D shapes, 3D figures • Mental transformations • Spatial language comprehension • Visual-spatial working memory
4. Shape Transformer 	• Modeled after the 'Function Machine,' an 'input/output' activity typically done with numbers (e.g., input = 2, output = 4; input = 5, output = 10, . . . etc. Rule, $y = 2x$) • In this version, input and output functions deal with spatial relationships (e.g., transformations) • Children were presented with a "machine" made out of a poster board, with "input" and an "output" slots cut out • Teacher (and eventually students) prepared input and output cards to enter and exit into/out of the "machine" • Children watched and paid attention to relationship between input and output cards and tried to predict the transformation (e.g., each shape that goes into the machine gets rotated 45°)	• Mental transformations/visualization • Visual-spatial reasoning/ deductive reasoning • Composition/decomposition of 2D shapes
5. Barrier Game 	• Children worked in pairs with a barrier (folder) in between them and each with their own building materials (e.g., pattern blocks or multi-link cubes) • One partner built a shape/figure and described how to build the shape/figure to his/her partner, who built according to the instructions provided • Children then compared their structures before reversing roles	• Spatial language • Visualization • Composing/decomposing 2D shapes/3D figures

Table 3.1: Examples of "rug activities" carried out in the experimental classrooms

in pre and post assessments of spatial language, visual-spatial geometric reasoning, 2D mental rotation, number knowledge, magnitude comparison, and a repeated measures control task assessing receptive vocabulary. While there was no significant difference in performance between the two groups at the beginning of the school year, there were remarkable differences at the end of the year. Compared to the control group, children who had participated in the spatial activities demonstrated widespread improvements on all of the spatial measures, including spatial language, 2D mental rotation, and visual-spatial geometric reasoning. And surprisingly, a significant difference emerged on a test of symbolic (i.e., Arabic digits) magnitude comparison; a test shown to be significantly related to children's arithmetic performance (Nosworthy et al., 2013). This finding was not expected, as the intervention did not explicitly focus on number development. This is a novel finding and one that suggests the possibility that early spatial instruction not only benefits children's spatial competencies but might also contribute to the development of early numeracy skills.

On a final note, as has been reported in other spatial intervention studies (Cakmak, Isiksal, & Koc, 2014), both the teachers and students reported high levels of enjoyment and engagement throughout the intervention. The teachers who led the intervention agreed that the spatial activities offered multiple entry points for their diverse learners, and furthermore, led to new insights into the potential for spatial reasoning to serve as an important foundation for mathematics learning.

Linking ideas

We are entering an exciting and promising era of spatial reasoning research. It is no longer enough to show that spatial reasoning and mathematics are related. The time has come to explain why the two are related, and furthermore, to mobilize and apply our existing knowledge in fruitful and long-lasting ways. To the latter point, we see early years education as an important place to begin such efforts. In this chapter, we shared a working hypothesis of how and why spatial reasoning and mathematics go hand-in-hand, paying particular attention to how spatial reasoning can provide an important foundation for mathematics learning. Indeed, there is now extensive evidence that spatial reasoning is malleable and can be improved in people of all ages and through a wide variety of training techniques. Although the majority of spatial training studies have been conducted in carefully controlled "lab" experiments, the educational implications of these findings are significant and potentially far-reaching. In terms of early years mathematics education, there is an accumulating body of intervention studies pointing to the importance of providing opportunities for high quality construction play (e.g., building blocks), puzzle play, drawing exercises, and paper folding. In moving forward, we urge psychologists and mathematics educators to work together in both the design and implementation of classroom-based spatial interventions.