Proofs in Practice

Going Beyond "Explain Your Work"

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Why Proofs?

The concept of proof is central in the study of mathematics; it allows mathematicians to test the implications of ideas and derive knowledge based on principles that have already been established (Martin & Guershon, 1989). A proof can be defined as a "sequence of logical statements, one implying another, which gives an explanation of why a given statement is true" (Stefanowicz, 2014, p. 10). Proofs can serve different functions: they can be used to verify a statement and test its truth; to explain it and provide insight into why it is true; to organize results in a systematic way; to discover and derive new results and communicate; and to transmit mathematical knowledge (Villiers, 1990).

What the OME Says About Proving

The Mathematics Curriculum includes Reasoning and Proving as one of the processes that supports effective learning in mathematics as it leads to deeper understanding of new concepts (Ministry of Education, 2005, p. 14). Yet, students do not have enough opportunities to engage with proofs in elementary and when they do, it is usually in the context of Euclidean geometry (Stylianides, 2016, p. 8).

Proofs in the Elementary Class

In elementary classrooms, a "proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics: (1) It uses statements accepted by the classroom community (*set of accepted statements*) that are true and available without further justification; (2) It employs forms of reasoning (*modes of argumentation*) that are valid and known to, or within the conceptual reach of, the classroom community; and (3) It is communicated with forms of expression (*modes of argument representation*) that are appropriate and known to, or within the conceptual reach of, the classroom community" (Stylianides, 2007, p. 291).

Proving Continuum

1 DESCRIBE

Look Fors
- ✓ Student can tell you how to **compute** the answer or gives one example where the statement is true.

2 EXPLAIN

Look Fors
- ✓ Student **attempts to explain** the solution or statement.
- ✓ Explanation is not always correct and student might rely on intuition.

3 CONVINCE

Look Fors
- ✓ Student **begins to present an argument** supporting their claim (not necessarily accurate).
- ✓ Student often uses phrases such as 'I think" to explain.

4 JUSTIFY

Look Fors
- ✓ Student's argument is **logically correct.**
- ✓ Student's uses phrases such as 'because', 'therefore', 'and so', etc to present their argument.

5 PROVE

Look Fors
- ✓ Student offers a clear, logical argument.
- ✓ Student **uses evidence**(math 'facts' or axioms and conjectures) to support their argument.

* Adapted from the NRICH Website found at https://nrich.maths.org/11336

Supporting Students' Proving Ability

NRich presents five strategies that can be used in elementary classrooms to support students' proving ability.

By explicitly teaching these strategies and engaging students in proving activities, we can support students proving and reasoning skills and move them forward along the continuum.

Proof by Counter-Example:

- Finding one example to show that a statement does not hold true.
- Statement: Prove that the sum of any two odd numbers is odd (odd + odd = odd).
- Counter Example: 3 + 3 = 6. Hence an odd number + an odd number is not always odd.

Proof by Logical Reasoning:

- A proof that includes every single step in the reasoning process.
- Statement: Prove that the sum of any two odd numbers is even (odd + odd = even).
- Since we know that an odd number when divided by 2 has a remainder of 1 then an odd number, a, plus an odd number, b, divided by two would be $a/2$ R1 + $b/2$ R1 which is the same as $(a + b)/2$ + R2. Since a remainder 2 is divisible by 2 then a + b is divisible by two and hence even by definition.

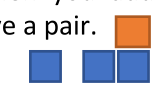
Proof by Exhaustion:

- consists of systematically finding all the possible outcomes.
- Statement: Prove that the sum of any 2 odd numbers less than 5 are even.

	1	3	5
1	2	4	6
3	4	6	8
5	6	8	10

Generic Proof:

- A carefully selected example that allows anyone to see the general structure of the argument.
- Statement: Prove that the sum of any two odd numbers is even (odd + odd = even).
- 1 + 3 can be represented with snap cubes:
- Since both numbers have ONLY one cube that does not have a pair, when you add them together, all cubes will have a pair.



- Since every odd number has this same format (one cube without a pair), this holds for every odd number.

Proof by Contradiction:

- Students assume that the statement is true and follow that logic until it leads them to a statement that contradicts this and/or is not possible.
- Statement: Prove that the sum of 2 odd numbers is even.
- Proof: Assume that the sum of 2 odd numbers is odd.
- Since we know that an odd number when divided by 2 has a remainder of 1 then an odd number, a, plus an odd number, b, divided by two would be a/2 R1 + b/2 R1 which is the same as (a +b)/2 + R2. Since a remainder 2 is divisible by 2 then a + b is divisible by two and hence even which is a contradiction to our assumption.

Useful Resources

NRich https://nrich.maths.org/11463

Stylianides, A (2016). Proving in the Elementary Mathematics Classroom. Oxford University Press.

Stylianides, A (2007). Proof and Proving in School Mathematics. Journal for Research in Mathematics Education, Vol. 38, No. 3 (May, 2007), pp. 289-321